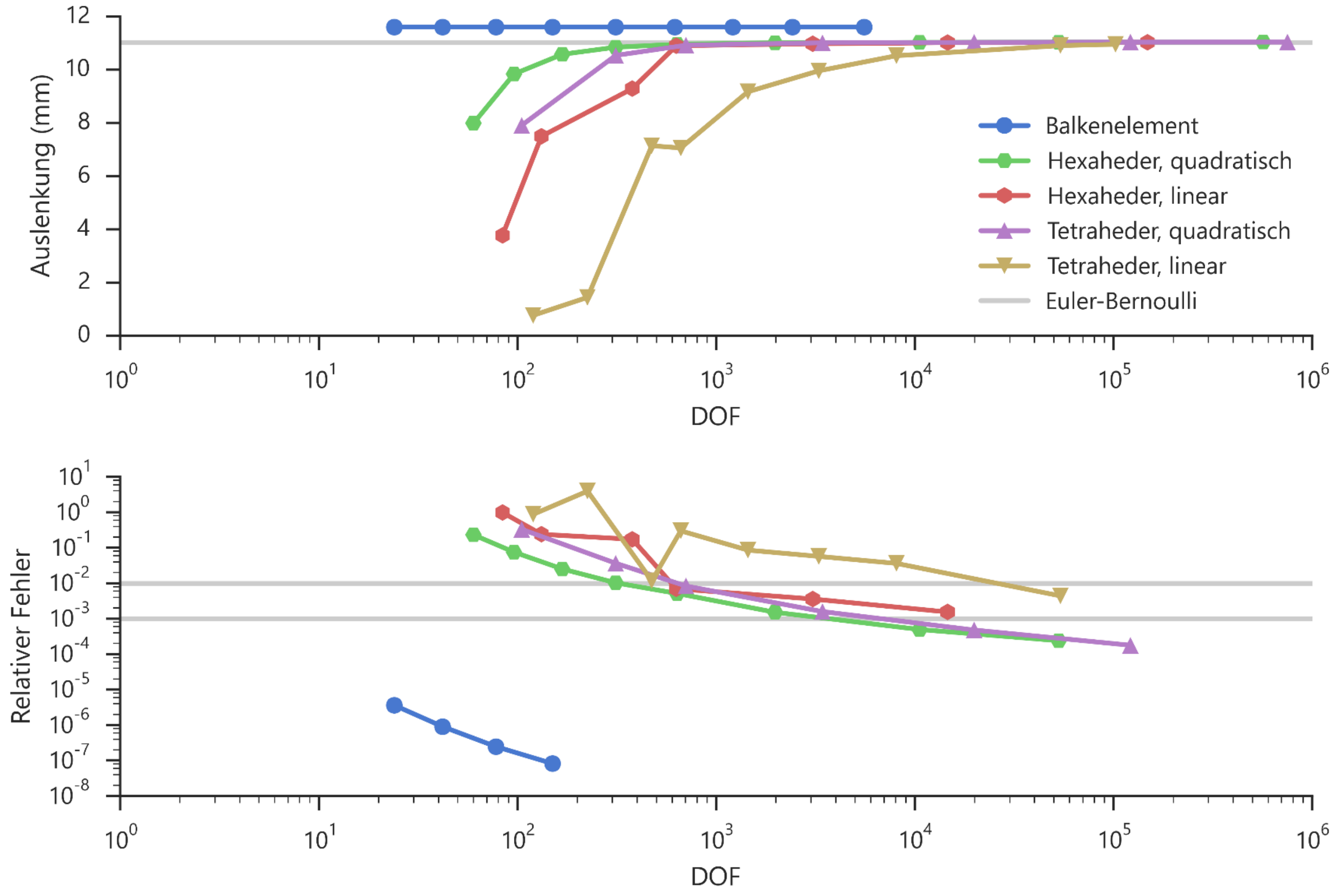
Ergebnisse

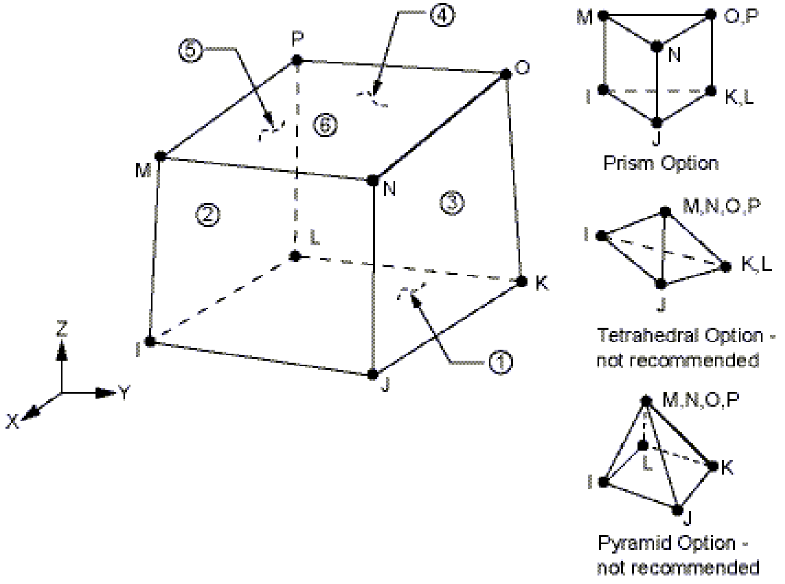
Balken	Auslenkung w_{max} (mm)						
Geometrie, Material	Experiment	Analytische Abschätzung	Numerische Simulation (konvergierte Lösung)				
			Tetraeder		Hexaeder		
			linear	quadratisch	linear	quadratisch	
Kunststoff	9	55			55 (13000)	55 (628)	
Holz	16	16,5	13,6 (> 250 000)	16	16,12	16,28 (3195)	
Alu	13	13,0	5,6	9,73 (1024)	22	9,74 (6600)	

Konvergenzanalyse (3D)

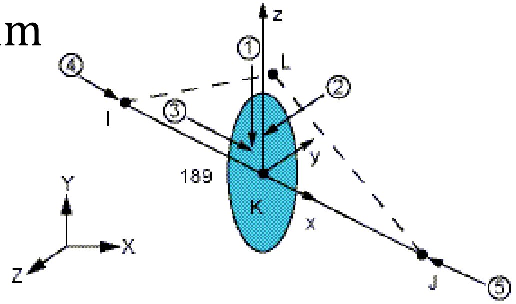


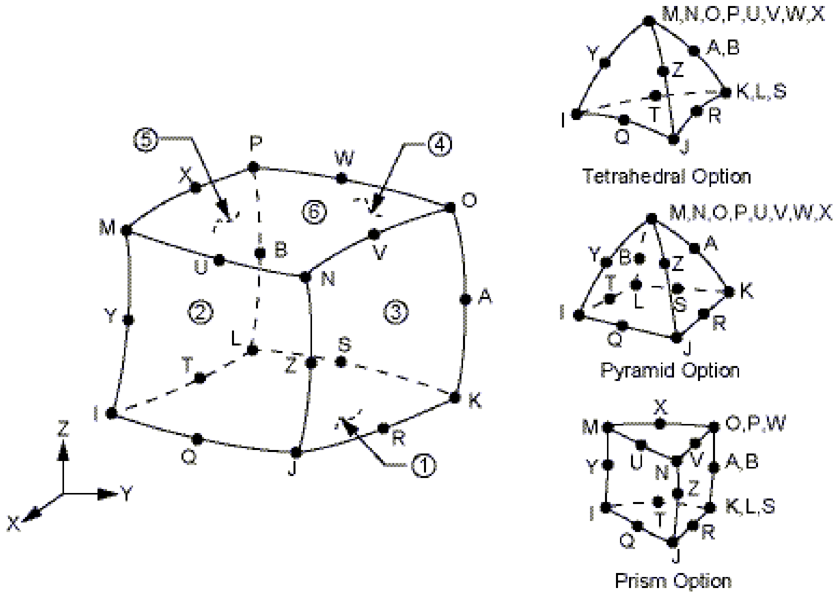
Konvergenzanalyse (3D)

Form	Ansatz	DOFs	Auslenkung (mm)
Tetraeder	linear	53922	10,91
Tetraeder	quadratisch	705	10,93
Hexaeder	linear	630	10,90
Hexaeder	quadratisch	312	10,85
Timoshenko-Balken	quadratisch	18	11,61



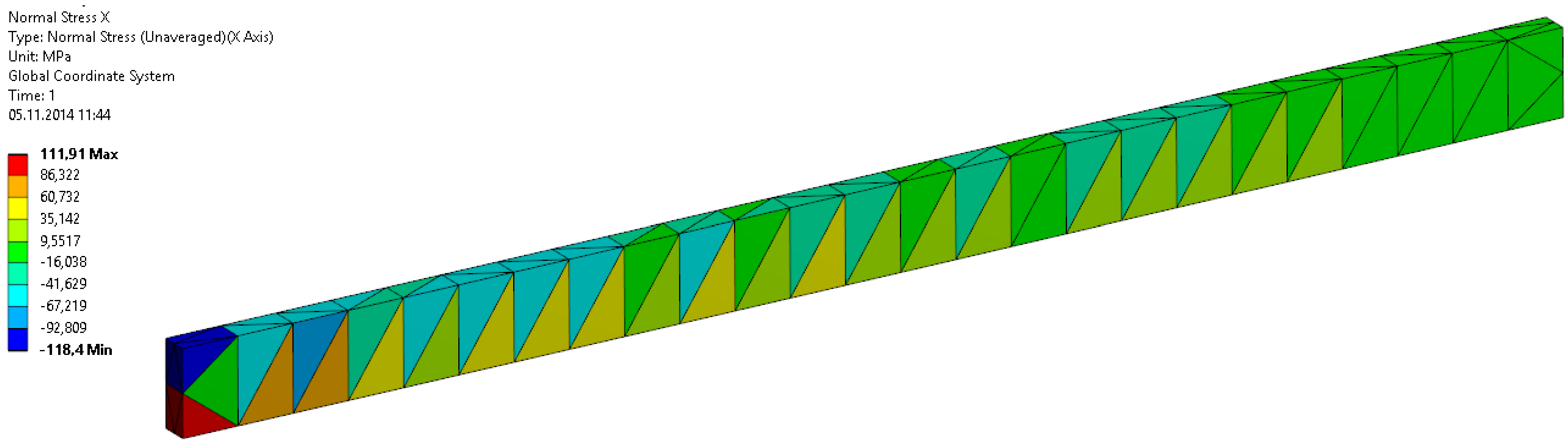
Euler-Bernoulli: $w_{\max} = \frac{Fl^3}{3EI} = 12\,\frac{Fl^3}{3Ebh^3} = 11{,}02\ \mathrm{mm}$



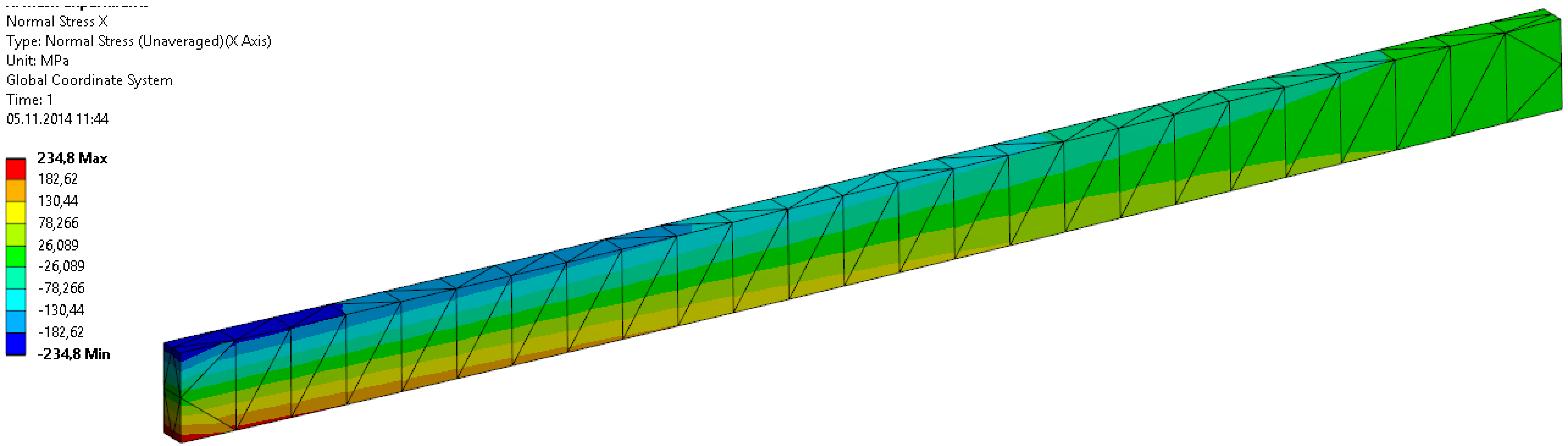


25x2x2 Tetraeder

Linear

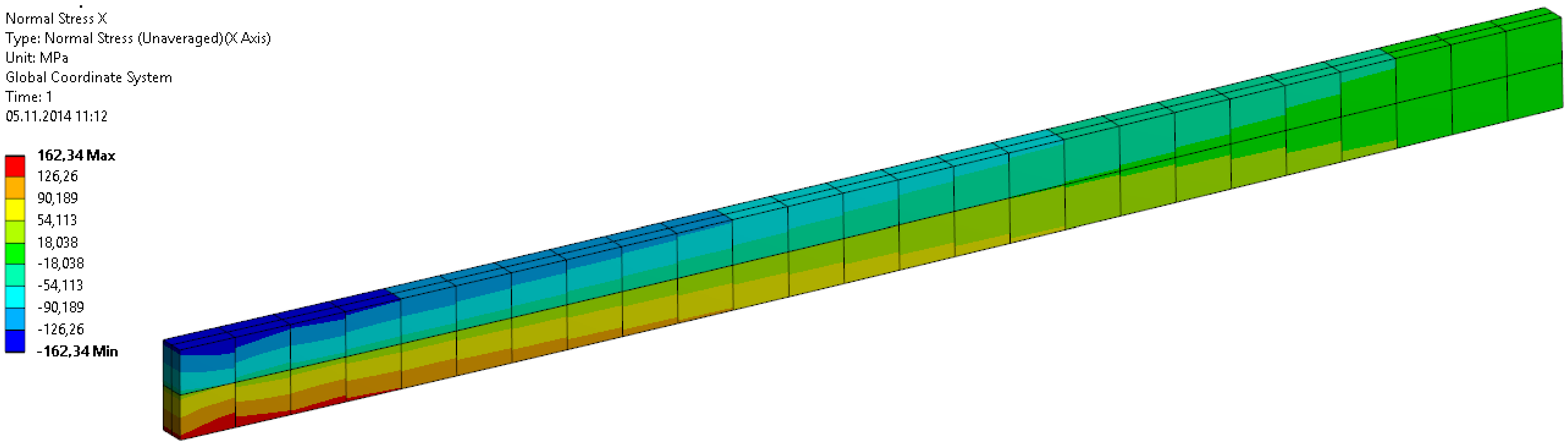


Quadratisch

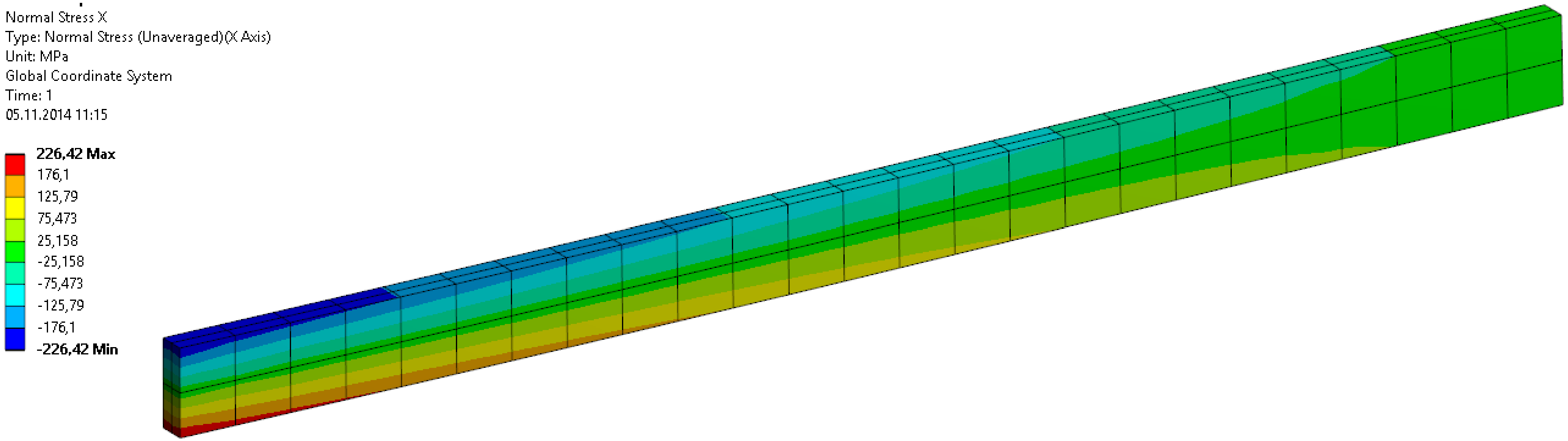


25x2x2 Hexaeder

Linear



Quadratisch



Shear Locking

Figure 4–4 Deformation of material subjected to bending moment *M*.

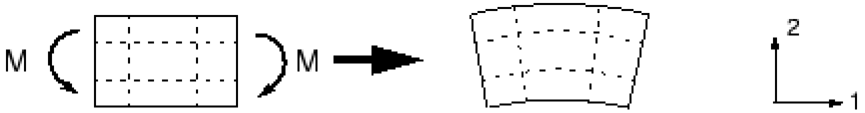


Figure 4–5 Deformation of a fully integrated, linear element subjected to bending moment *M*.

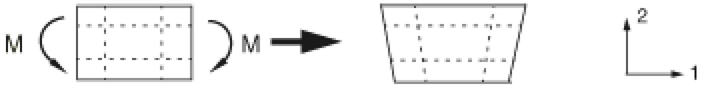


Figure 4–6 Deformation of a fully integrated, quadratic element subjected to bending moment *M*.

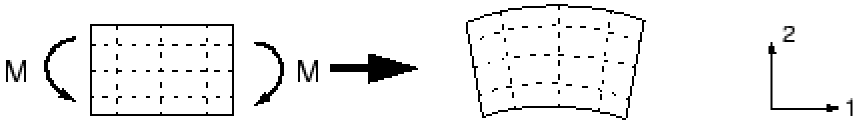


Figure 4–8 Deformation of a linear element with reduced integration subjected to bending moment *M*.

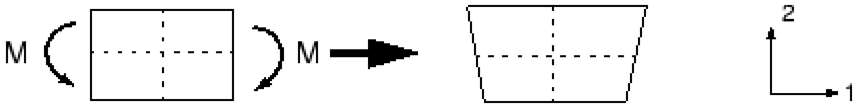


Figure 4–2 Integration points in fully integrated, two-dimensional, quadrilateral elements.

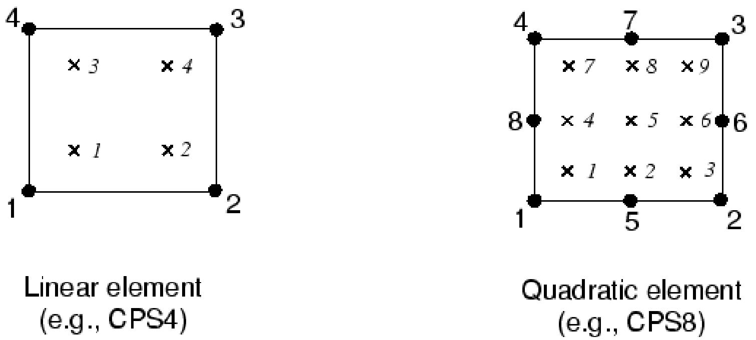
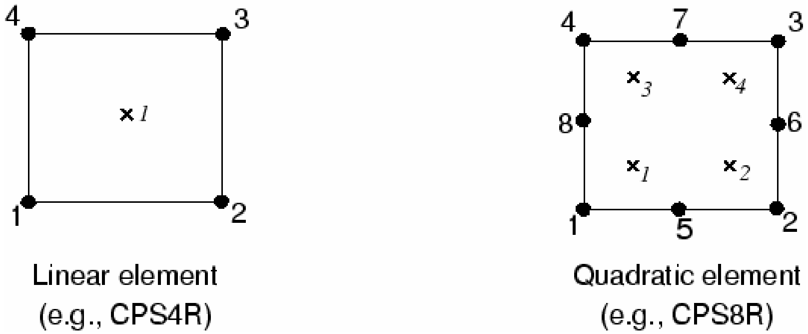


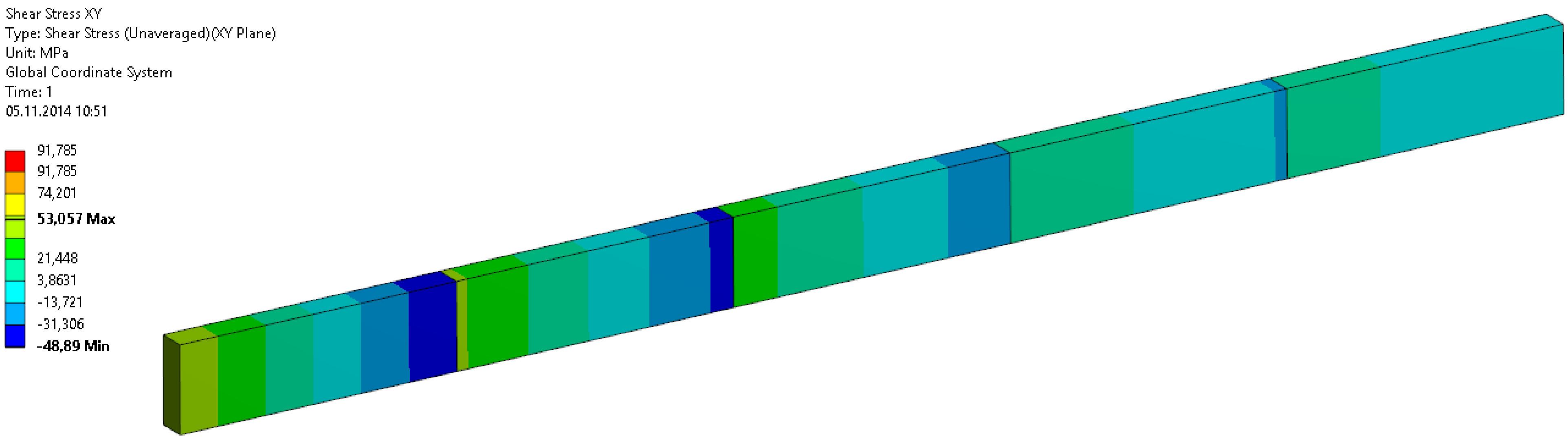
Figure 4–7 Integration points in two-dimensional elements with reduced integration.



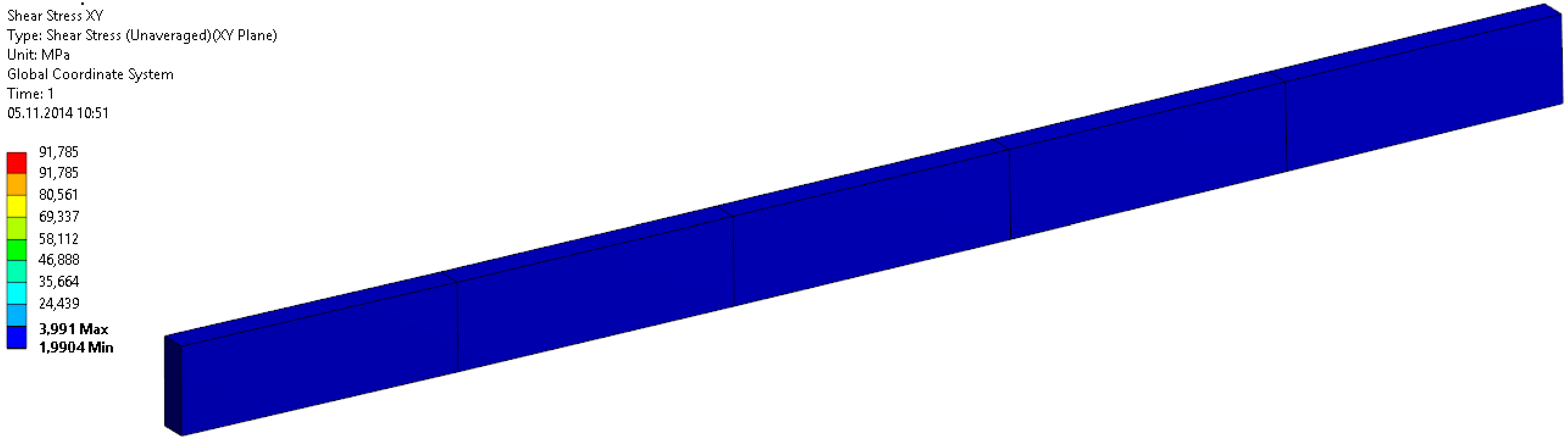
Abaqus-Manual, Kapitel 4

Shear Locking

Linear

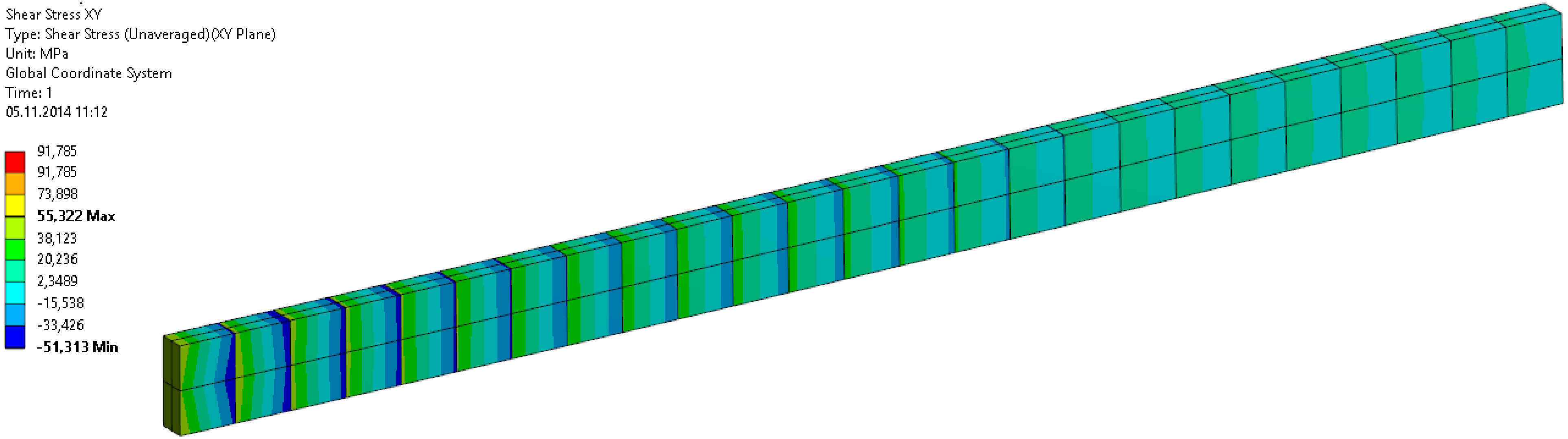


Quadratisch

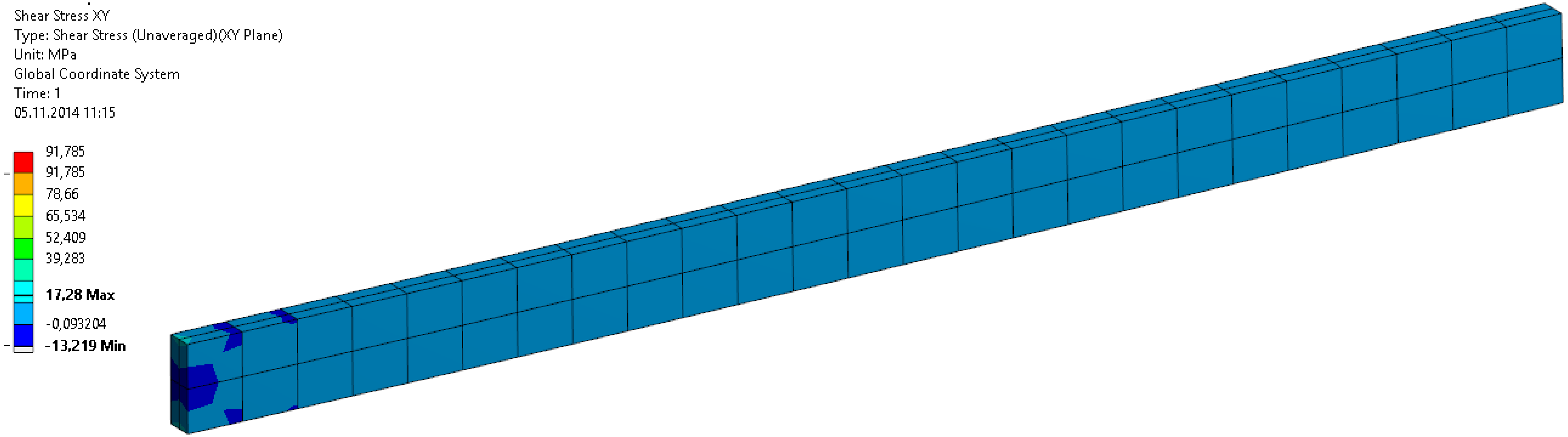


25x2x2 Hexaeder

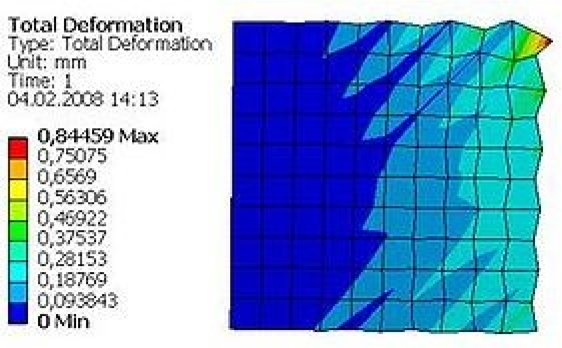
Linear



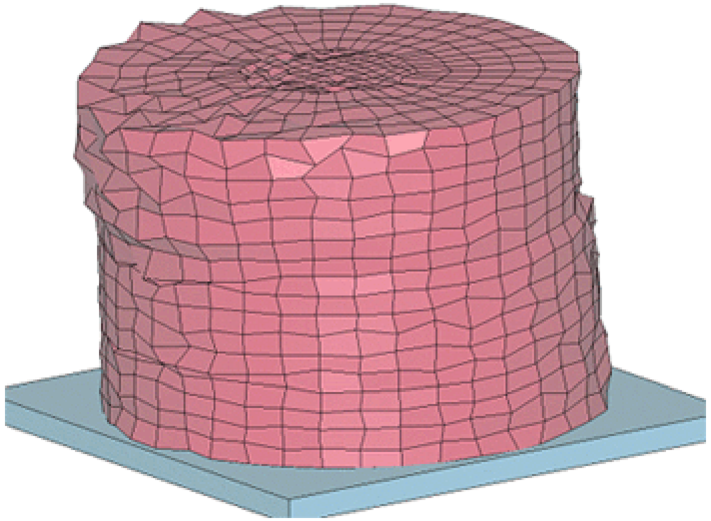
Quadratisch



Hourglassing

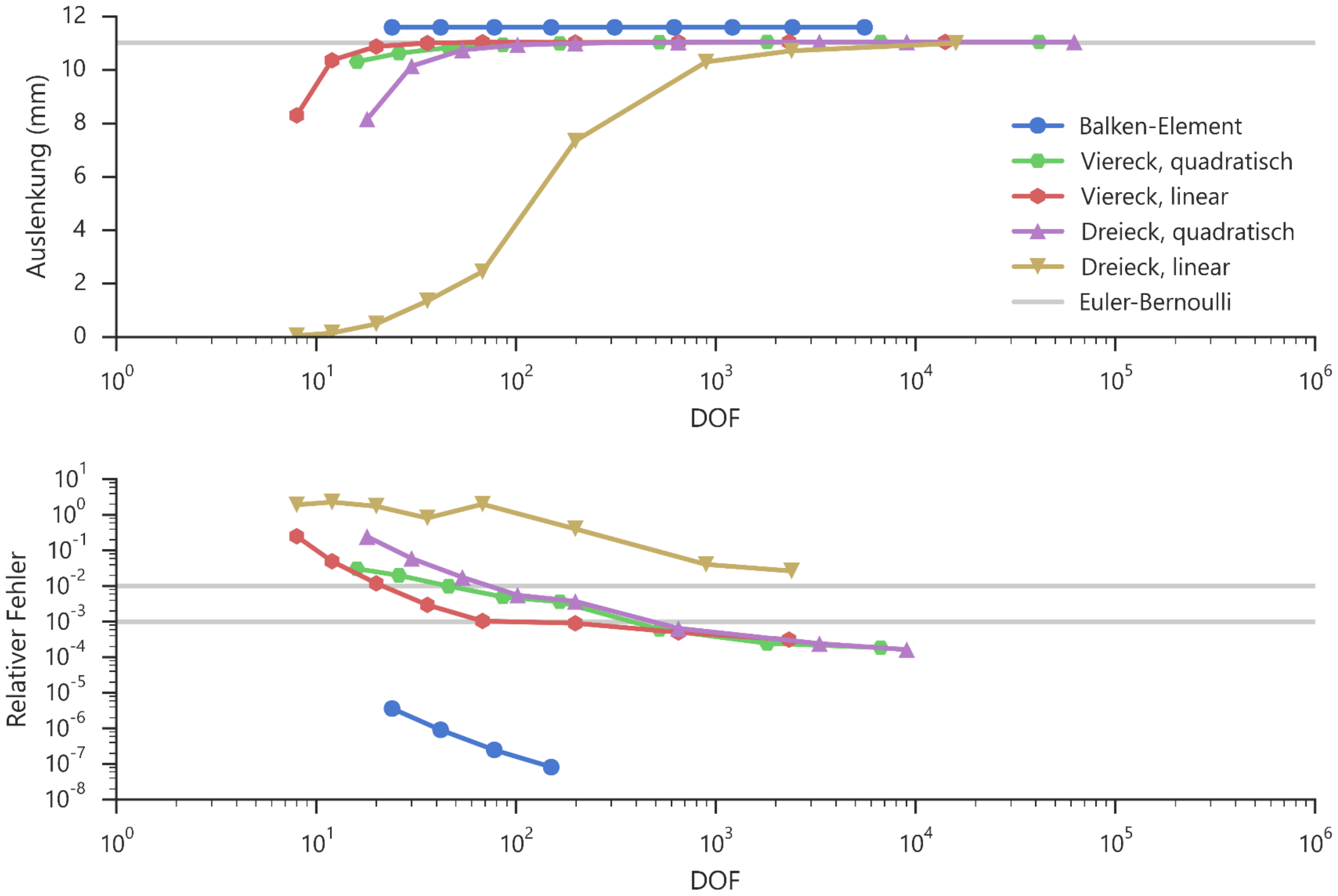


CADFEM Wiki



U.S. Federal Highway Administration

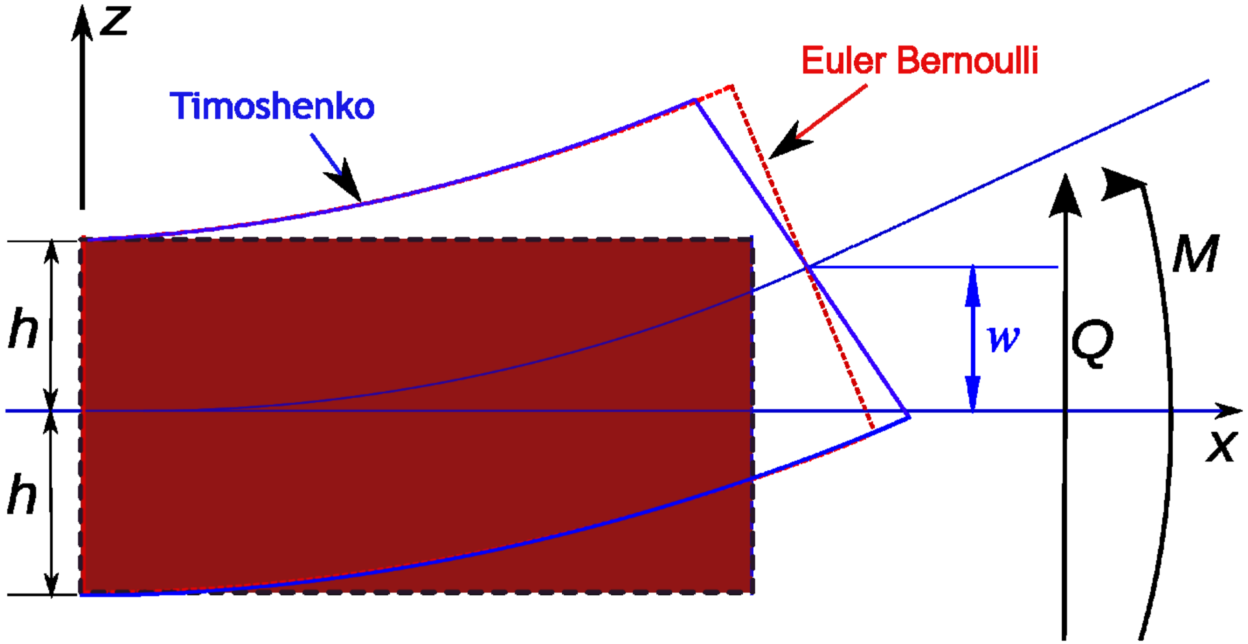
Konvergenzanalyse (2D)



Timoshenko vs. Euler-Bernoulli

*"The BEAM189 element is suitable for analyzing **slender to moderately stubby/thick** beam structures. The element is based on Timoshenko beam theory which **includes shear-deformation** effects."*

ANSYS 15.0 Documentation (Element Library)

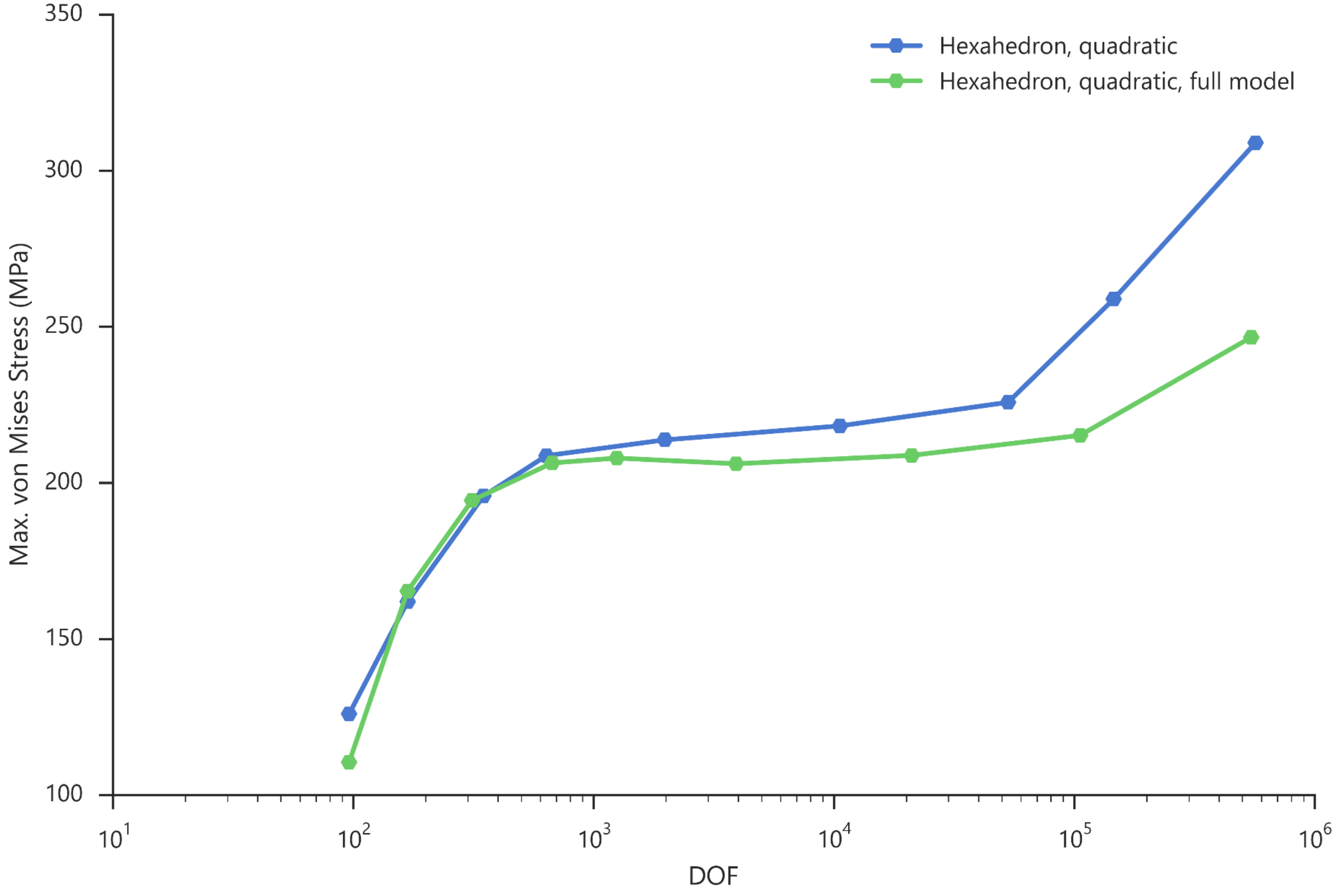


http://en.wikipedia.org/wiki/Timoshenko_beam_theory

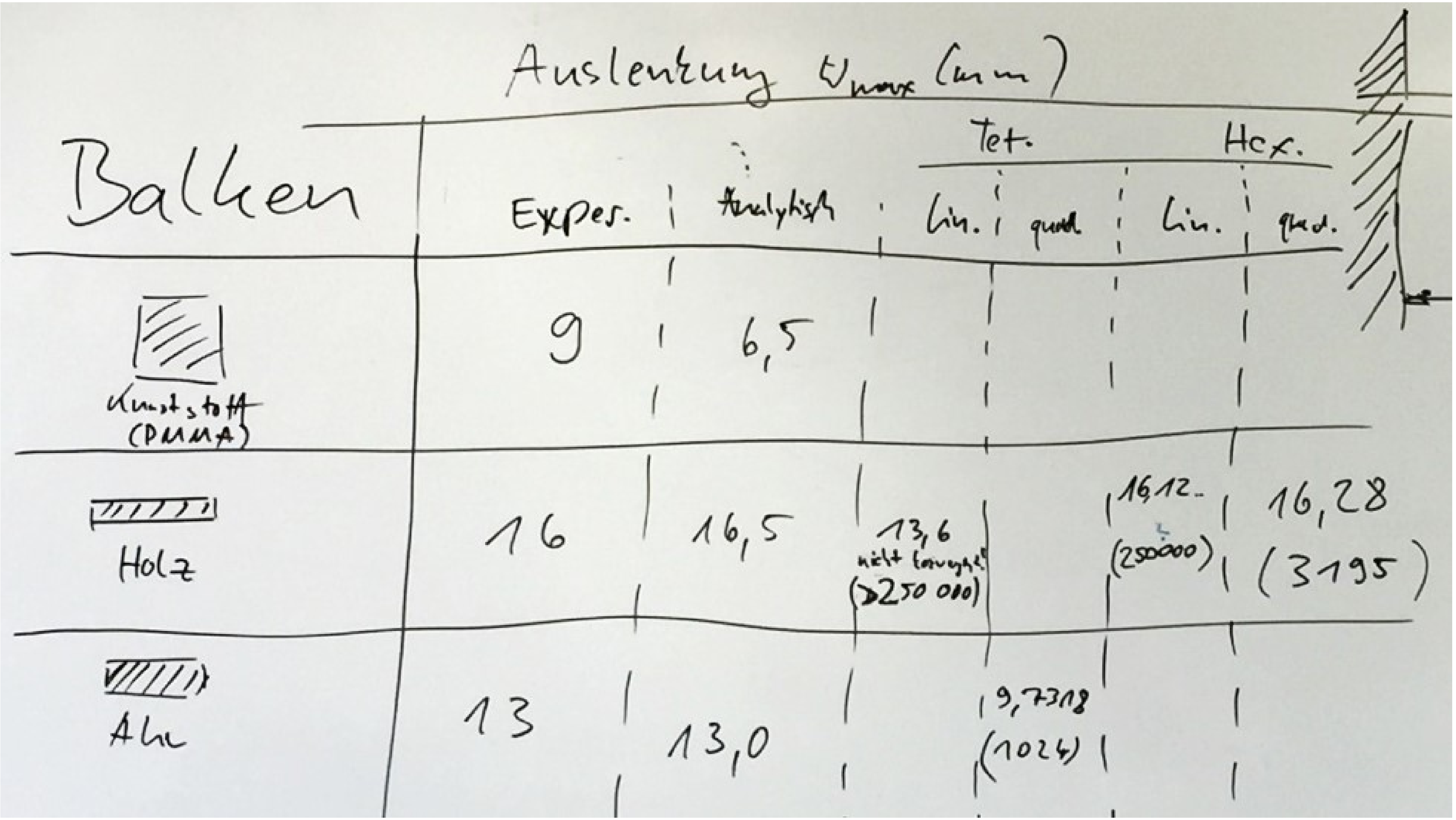
$$w_{\max}^{\mathrm{EB}} = \frac{Fl^3}{3EI}$$

$$w_{\max}^{\mathrm{T}} = \frac{Fl}{\kappa AG} + \frac{Fl^3}{3EI}$$

Konvergenzanalyse (3D)



Validierung



Zusammenfassung

- Lineare Tetraeder (und Dreieckselemente) – soweit möglich – vermeiden (zu steif)
- Vollintegrierte lineare Hexaeder sind zu steif unter Biegebeanspruchung (Shear Locking)
- Reduziert-integrierte lineare Hexaeder können zu Hourglassing führen
- Quadratische Hexaeder zeigen sehr gute Konvergenzeigenschaften und sind relativ unempfindlich in Bezug auf die Netzqualität, sind aber teuer (20 Knoten/Element)
- Quadratische Tetraeder bieten einen guten Kompromiß aus Genauigkeit und Praxistauglichkeit (jede Topologie ist damit vernetzbar)
- Für Balkenbiegeprobleme bietet sich die Verwendung spezieller Balkenelemente an
- 3D-Modellen benötigen um Größenordnungen mehr DOFs als 2D- und 1D-Modelle
- Bei der Validierung mit experimentellen Daten immer beachten:
    - Messungen sind **immer** fehlerbehaftet („Wer mißt, mißt Mist.“)!
    - Experimentelle Daten sind wichtig, sollten aber auch nicht überbewertet werden